Newton's Universal Law of Gravitation

Every object with mass attracts every other object with mass with a gravitational force.

This gravitational force is directly proportional to product of the masses of BOTH objects.

This gravitational force is inversely proportional to the square of the distance between the centers of the two objects.

$$F_G = G\frac{M_1 M_2}{d^2}$$

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- Gravity is a universal force
- There's a fancy equation we need to know about

Mass of object 1

Gravitational Constant

The force

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Mass of object 2

A distance apart from each other in meters (and squared)

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The force

Gravitational Constant

Mass of object 1

$$F = G\frac{m_1 m_2}{d^2}$$

Mass of object 2

A distance apart from each other in meters (and squared)

$$F = \frac{GMm}{R^2}$$

$m = 1\ kg$, $M = 6 \times 10^{24}\ kg.$

$R = 6.4 \times 10^{6}\ m$, $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$$\therefore F = \frac{6.67 \times 10^{-11} Nm^2/kg^2 \times 6 \times 10^{24} kg \times 1 kg}{(6.4 \times 10^6 m)^2}$$

$\therefore F =$ N

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$$\therefore \boxed{F = 9.8\ N}$$

9'77 N

Sample Problem #1
Newton's Law of Universal Gravitation

- **Question:** What is the gravitational force of attraction between two asteroids in space, each with a mass of 50,000 kg, separated by a distance of 3800 m?
- **Answer:** $F_g = \frac{Gm_1m_2}{r^2}$

50 000 kg

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$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_g = \frac{\left(6.67\times10^{-11}\ N\bullet m^2/kg^2\right)(50000kg)(50000kg)}{(3800m)^2}$$

$$F_g = 1.15\times10\ N$$

- As you can see, **the force of gravity is a relatively weak force**, and we would expect a relatively weak force between relatively small objects**. It takes tremendous masses and small distances in order to develop significant gravitational forces**. Let's take a look at another problem to explore the relationships between gravitational force, mass, and distance.

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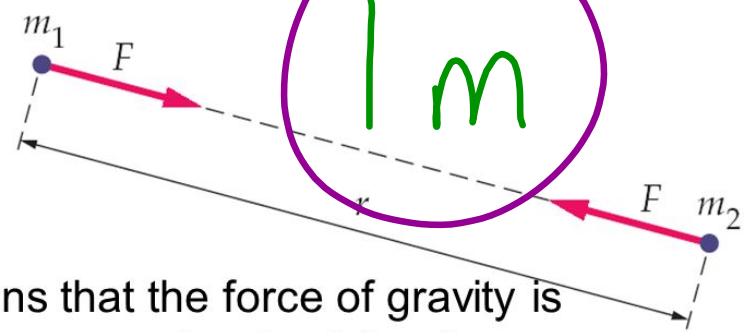
The gravitational force is always attractive, and points along the line connecting the two masses.

1 m

The two forces shown are an action-reaction pair.



G is a very small number; this means that the force of gravity is negligible unless there is a very large mass involved (such as the Earth).

Ex. How strong is gravitational attraction between you and the person next to you?

Solution:

Estimate: $m_1 \approx m_2 \approx 60\ kg,\ r_{12} \approx 1\ m$

$$\rightarrow F = \left(6.67\times10^{-11}\right)\frac{(60)(60)}{1^2} \approx \bullet \times 10^{\heartsuit}\ N$$

2 4

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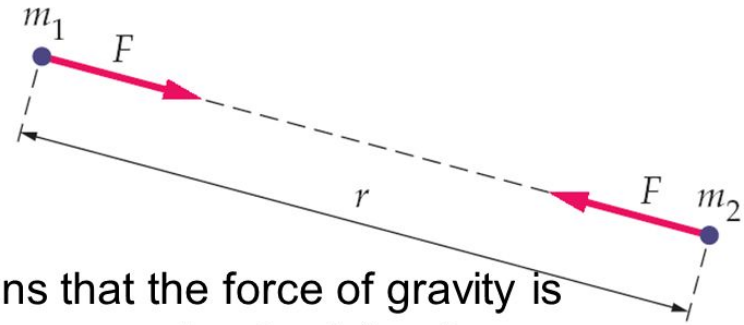

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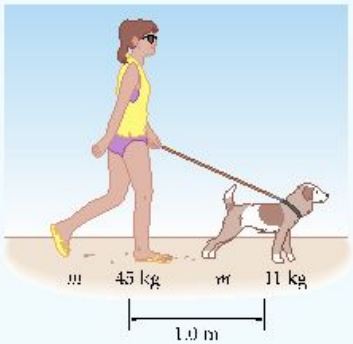
$$\rightarrow F = \left(6.67\times10^{-11}\right)\frac{(60)(60)}{1^2} \approx 2\times10^{-7}\ N$$

Newton's Law of Universal Gravity

- The example below illustrates how Newton's law of gravity may be applied. Note gravity's minuscule effects on ordinary objects.

QUICK Example 9.1 What's the Force?

A girl takes her dog for a walk on a sunny beach, as shown below. Find the force of gravity between the 45-kg girl and her 11-kg dog when they are separated by a distance of 1.0 m. (Treat the girl and her dog as point masses.)



$m = 45\,kg$

$m = 11\,kg$

$d = 1\,m$

Solution

Substitute $m_1 = 45$ kg, $m_2 = 11$ kg, and $r = 1.0$ m into $F = Gm_1m_2/r^2$:

$$F = G\frac{m_1m_2}{r^2}$$

$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(45\ \mathrm{kg})(11\ \mathrm{kg})}{(1.0\ \mathrm{m})^2}$$

$$= \boxed{}\ \mathrm{N}$$

This is roughly 3 ten-millionths of a newton—much too small to be noticed. (Though we chose $m_1 = 45$ kg and $m_2 = 11$ kg, the same answer is obtained with $m_1 = 11$ kg and $m_2 = 45$ kg.)

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$3'3 \cdot 10^{-}$ N

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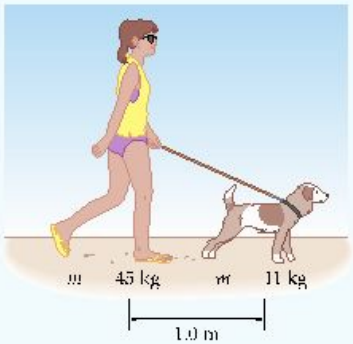

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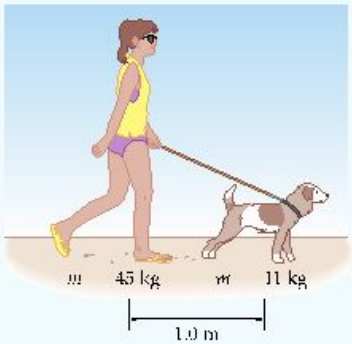

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Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67 \times 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(2.00\ \text{kg})(2.00\ \text{kg})}{(0.500\ \text{m})^2}$$

$$F_g = 1.07 \times 10^{\heartsuit}\ \text{N}$$

Statement: The magnitude of the gravitational force of attraction between the two spheres is $1.07 \times 10^{\heartsuit}$ N.

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$$F_g = 1.07 \times 10^{-9}\ \text{N}$$

Statement: The magnitude of the gravitational force of attraction between the two spheres is 1.07×10^{-9} N.

Find the gravitational force between the Jupiter and its natural satellite Callisto. The mass of the Jupiter is 1.99 X 10^{27}-kg and the mass of Callisto is 1.08 X 10^{23}-kg. The distance between the Jupiter and Callisto is 1.88 X 10^{9}-m.

$$F_G = G\frac{M_{Jupiter}M_{Callisto}}{d^2}$$

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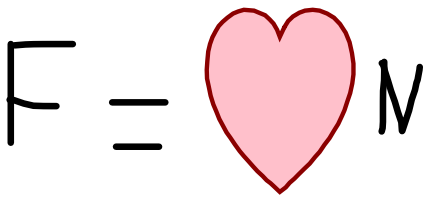

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$$F = 4'1\cdot10^{\heartsuit}\ N$$

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$$4'1\cdot10^{21}\ N$$

Ejercicio resuelto.

- **Calcular la atracción gravitatoria entre el Sol y la Tierra y las aceleraciones de cada astro.**
- Datos:

$$m_s = 2 \cdot 10^{30}\, kg\,;\, m_t = 6 \cdot 10^{24}\, kg\,;\, r_{s-t} = 1,5 \cdot 10^{8}\, km$$

- Usamos la ley de la gravitación universal (con unidades del SI)

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- Usamos la ley de la gravitación universal (con unidades del SI)

$$F = G\,\frac{m_s\cdot m_t}{r^2} = 6{,}67\cdot 10^{-11}\,\frac{N\cdot m^2}{kg^2}\cdot\frac{2\cdot 10^{30}\,kg\cdot 6\cdot 10^{24}\,kg}{(1{,}5\cdot 10^{11})^2\,m^2} = 3{,}6\cdot 10^{\heartsuit}\,N$$

$$3'6\cdot 10^{\heartsuit}\,N$$

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$3'6\cdot 10^{22}\,N$

Example problem

Now let's use Newton's law of universal gravitation to calculate the force of gravity here on Earth.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = 6.67\times10^{-11}\frac{5.98\times10^{24}\,m_2}{(6.37\times10^{6})^2}$$

$$F_g = 9.8m_2$$

As you can see Newton's law of universal gravitation is really another version of his second law of motion F=ma

$$F_{gravedad} = m\frac{GM_{Tierra}}{R^2_{Tierra}} = mg$$

$$M_E = 5{,}98x10^{24}kg$$

$$R_E = 6{,}38x10^6 m \quad (Medio)$$

$$G = 6{,}67259x10^{-11} Nm^2 / kg^2$$

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$9'8 \ \frac{N}{kg}$

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The gravitational force between these two objects is 100-N.

What will happen to the gravitational force if the red object's mass doubles?

$$F_G = G\frac{M_1 M_2}{d^2}$$

¿ F ?

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$$F_G = G\frac{M_1 M_2}{d^2}$$

Since only the mass of only one object changes, it is all that affects the change in the gravitational force.

$$2 \times M_1 \rightarrow 2 \times F_G$$

So doubling the mass also doubles the gravitational force:

$$F_{G_{NEW}} = 2 \times F_G = 2 \times 100 = \quad \text{N}$$

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So doubling the mass also doubles the gravitational force:

$$F_{G_{NEW}} = 2 \times F_G = 2 \times 100 = 200 \quad N$$

The gravitational force between these two objects is 100-N. What will happen to the gravitational force if the distance between them is cut in half?

$$F_G = G\frac{M_1 M_2}{d^2}$$

¿F?

The gravitational force between these two objects is 100-N. What will happen to the gravitational force if the distance between them is cut in half?

$$F_G = G\frac{M_1 M_2}{d^2}$$

Since only the distance changes, it is all that affects the change in the gravitational force.

$$\frac{1}{2}\times d \rightarrow \frac{F_G}{(\frac{1}{2})^2} \rightarrow \frac{F_G}{\frac{1}{4}} \rightarrow 4\times F_G$$

So halving the distance gets you four times the gravitational force:

$$F_{G_{NEW}} = 4\times F_G = 4\times 100 = \quad \text{N}$$

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So halving the distance gets you four times the gravitational force:

$$F_{G_{NEW}} = 4\times F_G = 4\times 100 = 400 \quad N$$

1 The radius of the Earth is 6.4×10^6 m and the gravitational field strength at its surface is 10 N kg^{-1}.
At what height above the surface of the Earth is the gravitational field strength equal to 2.5 N kg^{-1}?

2 Two 2.5 kg masses are placed with their centres 10 cm apart.
Calculate the size of the gravitational attractive force between them.
$G = 6.7 \times 10^{-11}$ N m^2 kg^{-2}.

3 The mass of the Moon is 7.4×10^{22} kg and its radius is 1.7×10^6 m.
Using the value of G from Q2, calculate the value of free-fall acceleration at the Moon's surface.

1 6.4×10^6 m
2 4.2×10^{-8} N
3 1.7 m s^{-2}